CS6375: Machine Learning

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Recurrent Neural Networks

Slides by Arun Mallya, Fei-Fei Li, Justin Johnson, Serena Yeung,
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Motivation

- Not all problems can be converted into one with fixed-length inputs and outputs

- Problems such as Speech Recognition or Time-series Prediction require a system to store and use context information
  - Simple case: Output YES if the number of 1s is even, else NO
    1000010101 – YES, 100011 – NO, …

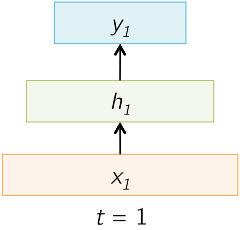
- Hard/Impossible to choose a fixed context window
  - There can always be a new sample longer than anything seen

Recurrent Neural Networks (RNNs)

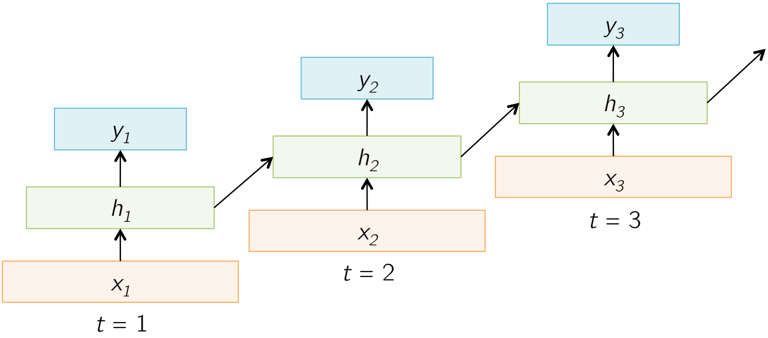
- **R**ecurrent **N**eural **N**etwork**s** take the previous output or hidden states as inputs.
  The composite input at time t has some historical information about the happenings at time $T < t$

- RNNs are useful as their intermediate values (state) can store information about past inputs for a time that is not fixed a priori

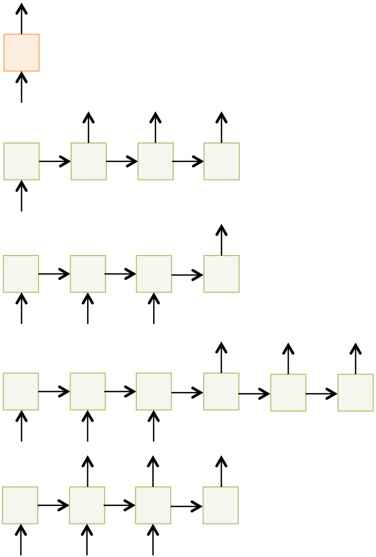
Sample Feed-forward Network



Sample RNN

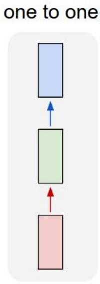


Input – Output Scenarios



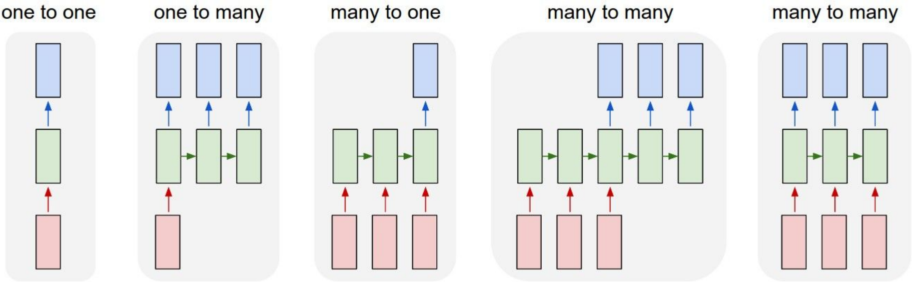
Single - Single	Feed-forward Network
Single - Multiple	Image Captioning
Multiple - Single	Sentiment Classification
Multiple - Multiple	Translation
	Image Captioning

“Vanilla” Neural Network



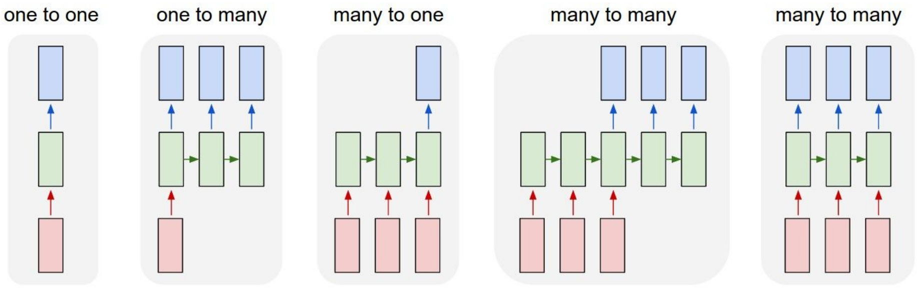
Vanilla Neural Networks

Recurrent Neural Networks: Process Sequences



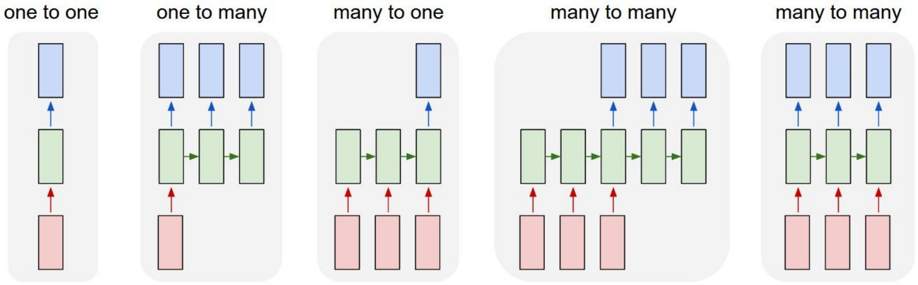
e.g. **Image Captioning**
image -> sequence of words

Recurrent Neural Networks: Process Sequences



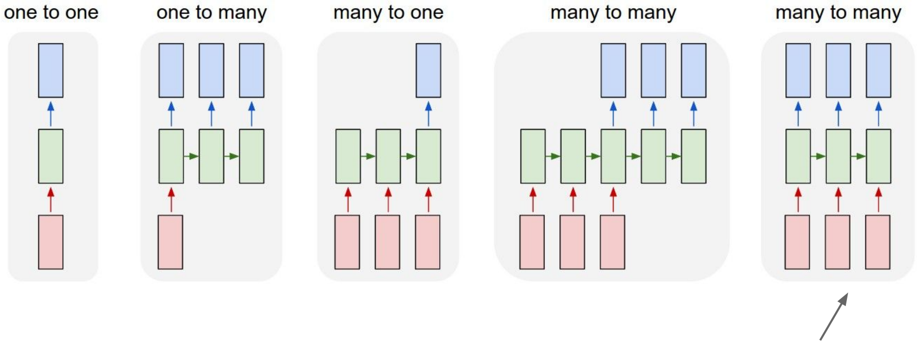
e.g. **Sentiment Classification**
sequence of words -> sentiment

Recurrent Neural Networks: Process Sequences



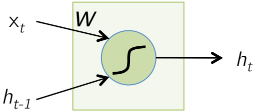
e.g. **Machine Translation**
seq of words -> seq of words

Recurrent Neural Networks: Process Sequences



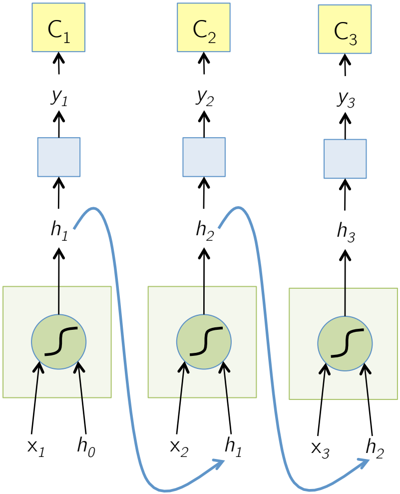
e.g. **Video classification on frame level**

The Vanilla RNN Cell



$$h_t = \tanh W \begin{pmatrix} x_t \\ h_{t-1} \end{pmatrix}$$

The Vanilla RNN Forward



$$h_t = \tanh W \begin{pmatrix} x_t \\ h_{t-1} \end{pmatrix}$$

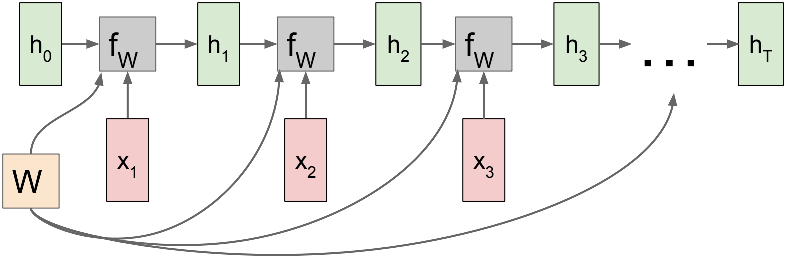
$$y_t = \mathrm{F}(h_t)$$

$$C_t = \mathrm{Loss}(y_t, \mathrm{GT}_t)$$

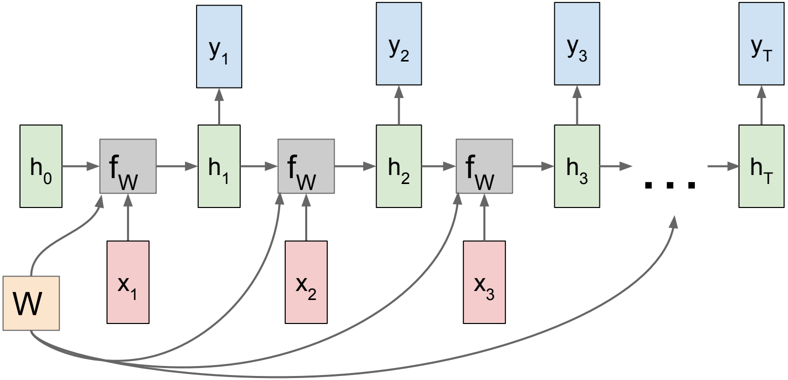
"Unfold" network through time by making copies at each time-step

RNN: Computational Graph

Re-use the same weight matrix at every time-step



RNN: Computational Graph: Many to Many



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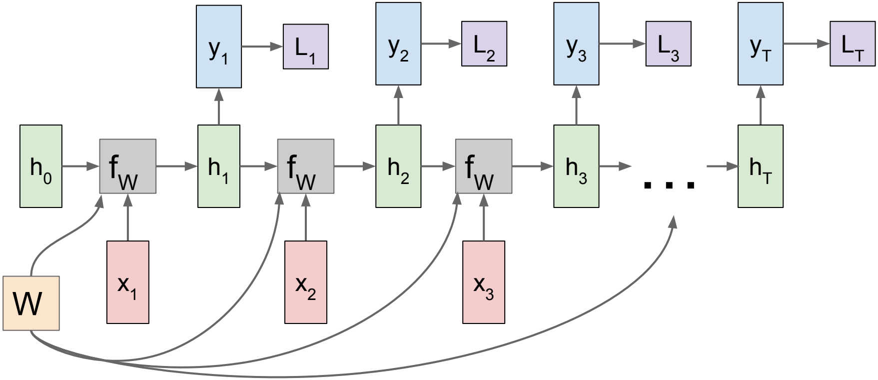

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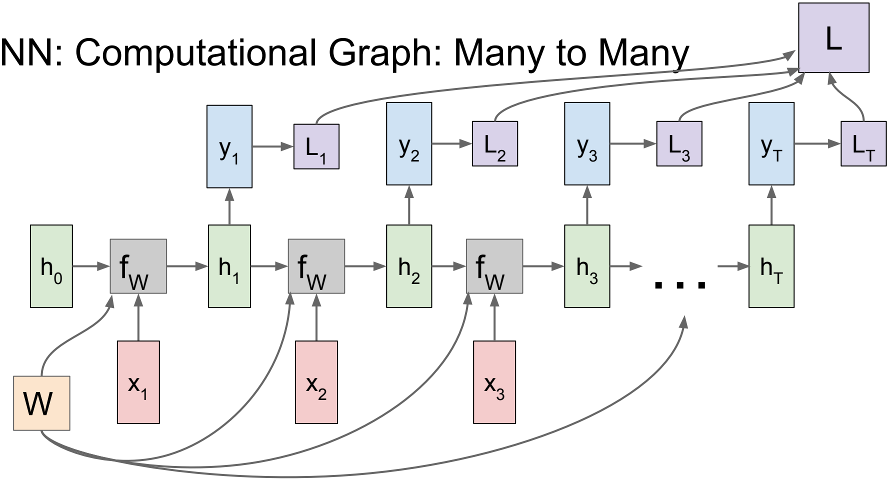
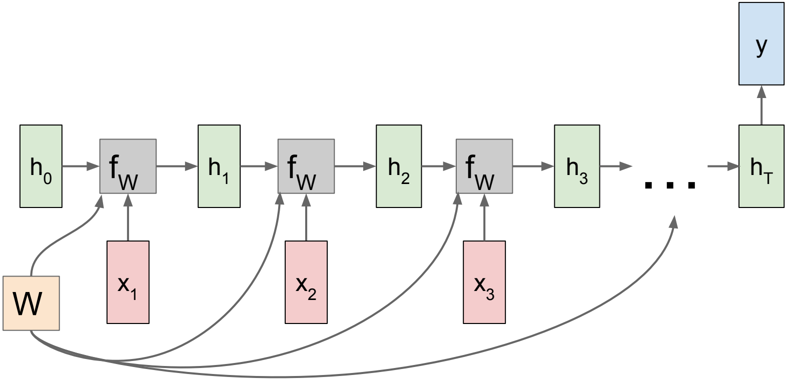
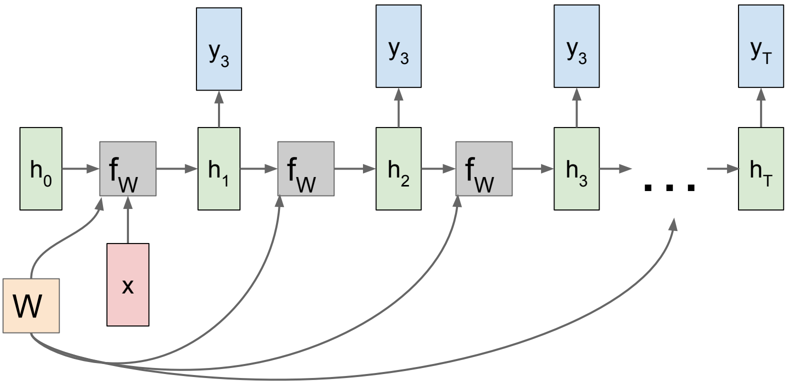


RNN: Computational Graph: Many to One

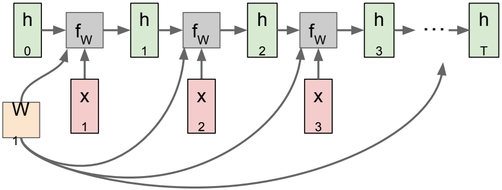


RNN: Computational Graph: One to Many



Sequence to Sequence: Many-to-one + one-to-many

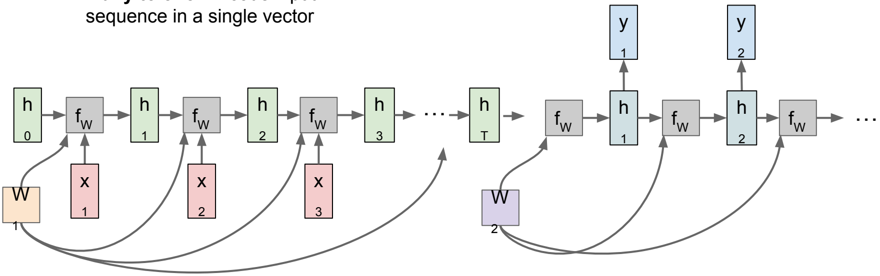
Many to one: Encode input sequence in a single vector



Sequence to Sequence: Many-to-one + one-to-many

One to many: Produce output sequence from single input vector

Many to one: Encode input sequence in a single vector



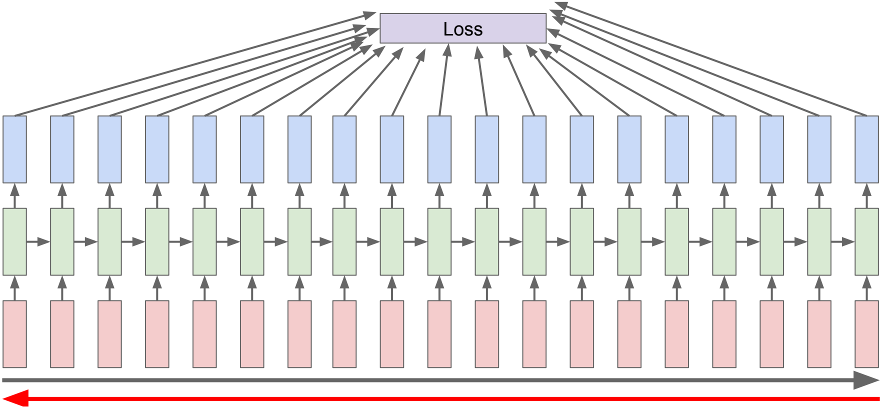
BackPropagation Through Time (BPTT)

- One of the methods used to train RNNs
- The unfolded network (used during forward pass) is treated as one big feed-forward network
- This unfolded network accepts the whole time series as input

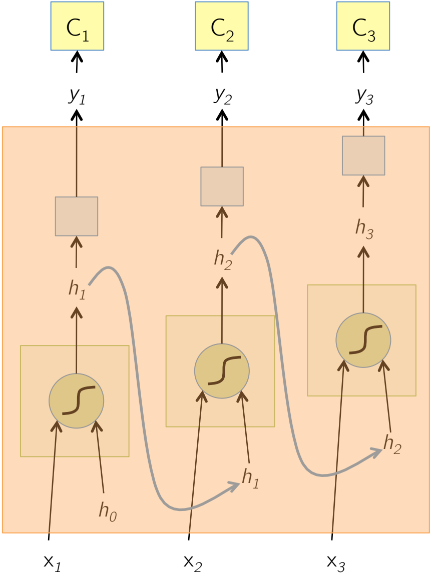
- The weight updates are computed for each copy in the unfolded network, then summed (or averaged) and then applied to the RNN weights

Backpropagation through time

Forward through entire sequence to compute loss, then backward through entire sequence to compute gradient

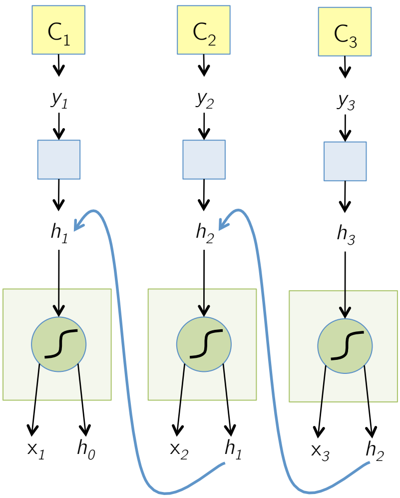


The Unfolded Vanilla RNN



- Treat the unfolded network as one big feed-forward network!
- This big network takes in entire sequence as an input
- Compute gradients through the usual backpropagation
- Update shared weights

The Vanilla RNN Backward



$$h_t = \tanh W \begin{pmatrix} x_t \\ h_{t-1} \end{pmatrix}$$

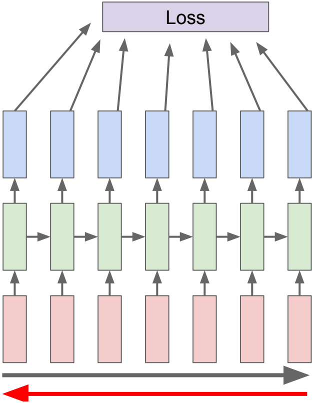
$$y_t = \mathrm{F}(h_t)$$

$$C_t = \mathrm{Loss}(y_t, \mathrm{GT}_t)$$

$$\frac{\partial C_t}{\partial h_1} = \left(\frac{\partial C_t}{\partial y_t}\right)\left(\frac{\partial y_t}{\partial h_1}\right)$$

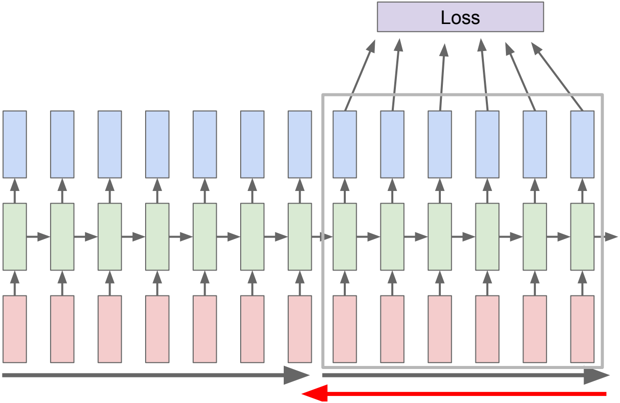
$$= \left(\frac{\partial C_t}{\partial y_t}\right)\left(\frac{\partial y_t}{\partial h_t}\right)\left(\frac{\partial h_t}{\partial h_{t-1}}\right)\cdots\left(\frac{\partial h_2}{\partial h_1}\right)$$

Truncated Backpropagation through time



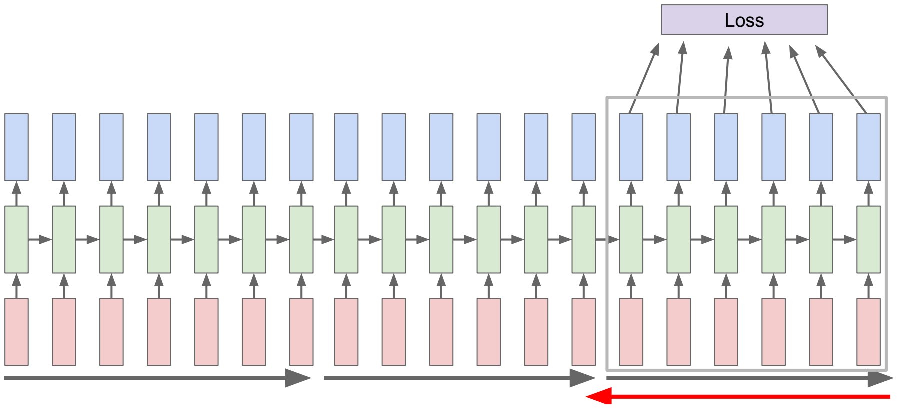
Run forward and backward through chunks of the sequence instead of whole sequence

Truncated Backpropagation through time



Carry hidden states forward in time forever, but only backpropagate for some smaller number of steps

Truncated Backpropagation through time



Issues with the Vanilla RNNs

- In the same way a product of k real numbers can shrink to zero or explode to infinity, so can a product of matrices

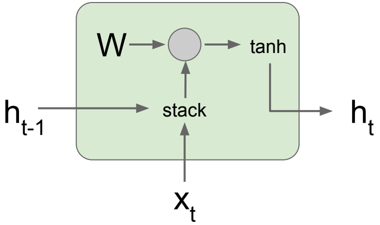
- It is sufficient for $\lambda_1 < 1/\gamma$, where λ_1 is the largest singular value of W, for the **vanishing gradients** problem to occur and it is necessary for **exploding gradients** that $\lambda_1 > 1/\gamma$, where $\gamma = 1$ for the tanh non-linearity and $\gamma = 1/4$ for the sigmoid non-linearity [1]

- Exploding gradients are often controlled with gradient element-wise or norm clipping

[1] On the difficulty of training recurrent neural networks, Pascanu *et al.*, 2013

Vanilla RNN Gradient Flow

Bengio et al, “Learning long-term dependencies with gradient descent is difficult”, IEEE Transactions on Neural Networks, 1994
Pascanu et al, “On the difficulty of training recurrent neural networks”, ICML 2013



$$
\begin{aligned}
h_t &= \tanh(W_{hh}h_{t-1} + W_{xh}x_t) \\
&= \tanh\left(\begin{pmatrix} W_{hh} & W_{hx} \end{pmatrix} \begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix}\right) \\
&= \tanh\left(W \begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix}\right)
\end{aligned}
$$

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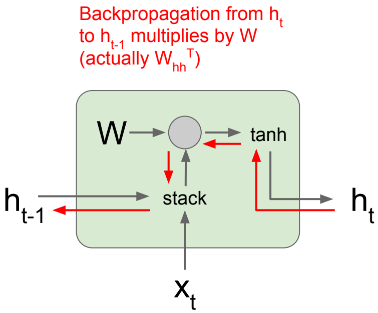

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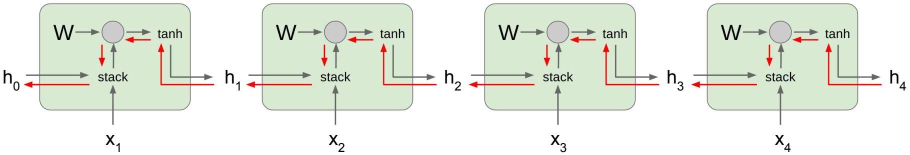

Computing gradient of h_0 involves many factors of W (and repeated tanh)

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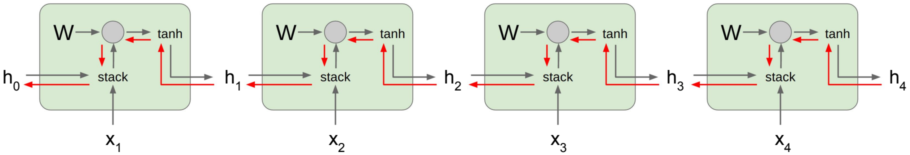

Computing gradient of h_0 involves many factors of W (and repeated tanh)

Largest singular value > 1:
Exploding gradients

Largest singular value < 1:
Vanishing gradients

Vanilla RNN Gradient Flow

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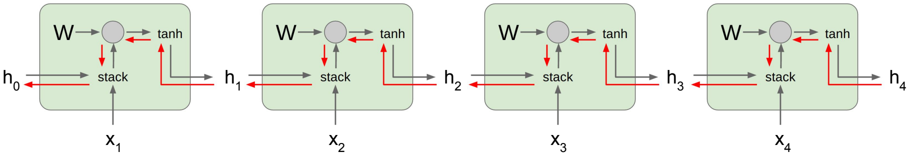
Pascanu et al, "On the difficulty of training recurrent neural networks", ICML 2013



Computing gradient of h_0 involves many factors of W (and repeated tanh)

Largest singular value > 1:
Exploding gradients

Largest singular value < 1:
Vanishing gradients

→ **Gradient clipping**: Scale gradient if its norm is too big

```
grad_norm = np.sum(grad * grad)
if grad_norm > threshold:
  grad *= (threshold / grad_norm)
```

Vanilla RNN Gradient Flow

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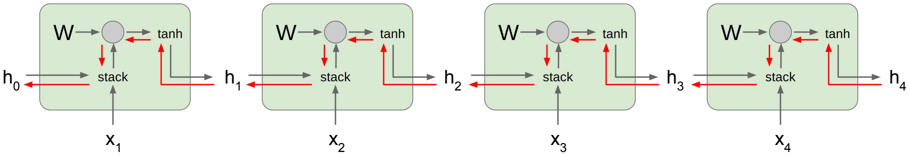


Computing gradient of h_0 involves many factors of W (and repeated tanh)

Largest singular value > 1:
Exploding gradients

Largest singular value < 1:
Vanishing gradients

→ Change RNN architecture

The Identity Relationship

- Recall $$\frac{\partial C_t}{\partial h_1} = \left(\frac{\partial C_t}{\partial y_t}\right)\left(\frac{\partial y_t}{\partial h_1}\right)$$ $$= \left(\frac{\partial C_t}{\partial y_t}\right)\left(\frac{\partial y_t}{\partial h_t}\right)\left(\frac{\partial h_t}{\partial h_{t-1}}\right)\cdots\left(\frac{\partial h_2}{\partial h_1}\right)$$

$$h_t = \tanh W \begin{pmatrix} x_t \\ h_{t-1} \end{pmatrix}$$

$$y_t = \mathrm{F}(h_t)$$

$$C_t = \mathrm{Loss}(y_t, \mathrm{GT}_t)$$

- Suppose that instead of a matrix multiplication, we had an **identity relationship** between the hidden states

$$h_t = h_{t-1} + F(x_t)$$

$$\Rightarrow \left(\frac{\partial h_t}{\partial h_{t-1}}\right) = 1$$

- The gradient does not decay as the error is propagated all the way back aka "Constant Error Flow"

Long Short-Term Memory (LSTM)[1]

- The LSTM uses this idea of "Constant Error Flow" for RNNs to create a "Constant Error Carousel" (CEC) which ensures that gradients don't decay

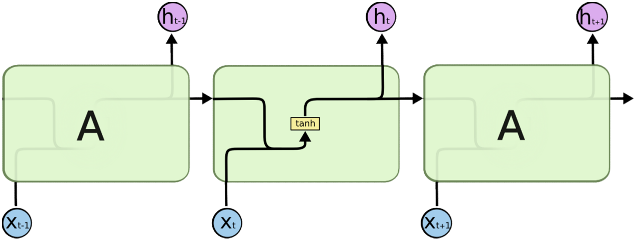
- The key component is a memory cell that acts like an accumulator (contains the identity relationship) over time

- Instead of computing new state as a matrix product with the old state, it rather computes the difference between them. Expressivity is the same, but gradients are better behaved

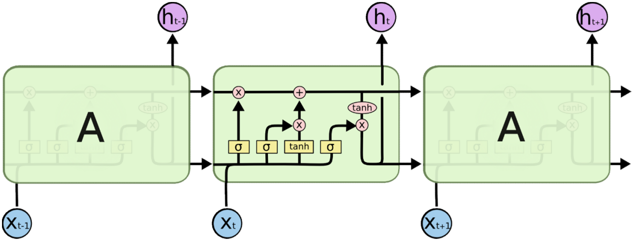
[1] Long Short-Term Memory, Hochreiter *et al.*, 1997

Long Short-Term Memory (LSTM) Networks

All **recurrent neural networks** have the form of a **chain of repeating modules**. In standard RNNs, this repeating module will have a very simple structure, such as a single tanh layer.



LSTMs also have this **chain-like structure**, but the repeating module has a different structure. Instead of having a single neural network layer, there are **four gates**, interacting in a very special way

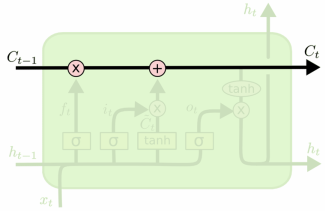


Figures and notes by Christopher Olah

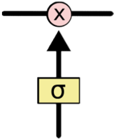
Long Short-Term Memory (LSTM) Networks

The key to LSTMs is the **cell state**, the horizontal line running through the top of the diagram.

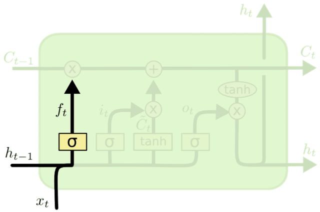
The cell state is like a conveyor belt. It runs straight down the entire chain, with only some minor linear interactions. It's very easy for **information to just flow along it uninterrupted**.



The LSTM has the ability to add or remove or add information to the cell state with structures called **gates**. Gates are composed out of a sigmoid neural net layer and a pointwise multiplication operation.



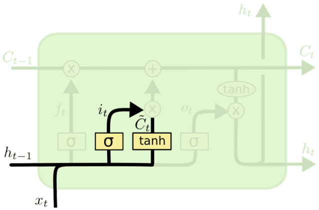
Forget Gate



$$f_t = \sigma\left(W_f \cdot [h_{t-1}, x_t] \ + \ b_f\right)$$

The first step in the LSTM cell is to decide what information we're going to throw away from the cell state. This decision is made by a sigmoid layer called the **forget gate layer**

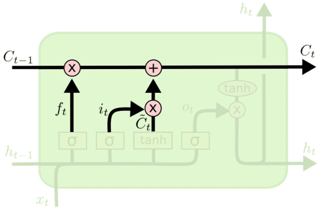
Input Gate



$$i_t = \sigma\left(W_i \cdot [h_{t-1}, x_t] \ + \ b_i\right)$$
$$\tilde{C}_t = \tanh(W_C \cdot [h_{t-1}, x_t] \ + \ b_C)$$

The next step is to decide what new information we're going to store in the cell state. This has two parts: the **input gate layer** decides which values to update, the **update layer** decides how much to update them by

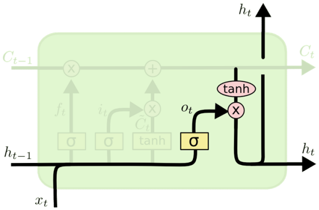
Update the Cell State



$$C_t = f_t * C_{t-1} + i_t * \tilde{C}_t$$

Simply **combine the outputs of the forget and input gates** to determine how much information this cell adds to or removes from the **"carousel"**

Output Gate



$$o_t = \sigma\left(W_o \ [h_{t-1}, x_t] \ + \ b_o\right)$$

$$h_t = o_t * \tanh\left(C_t\right)$$

This output will be based on our cell state, but will be a **filtered version**.

Summary

- RNNs allow for processing of variable length inputs and outputs by maintaining state information across time steps
- Various Input-Output scenarios are possible (Single/Multiple)

- Vanilla RNNs are improved upon by LSTMs which address the vanishing gradient problem through the CEC
- Exploding gradients are handled by gradient clipping

- More complex architectures are listed in the course materials for you to read, understand, and present

Other Useful Resources / References

- http://cs231n.stanford.edu/slides/winter1516_lecture10.pdf
- http://www.cs.toronto.edu/~rgrosse/csc321/lec10.pdf

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- S. Hochreiter, and J. Schmidhuber, Long short-term memory, Neural computation, 1997 9(8), pp.1735-1780
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